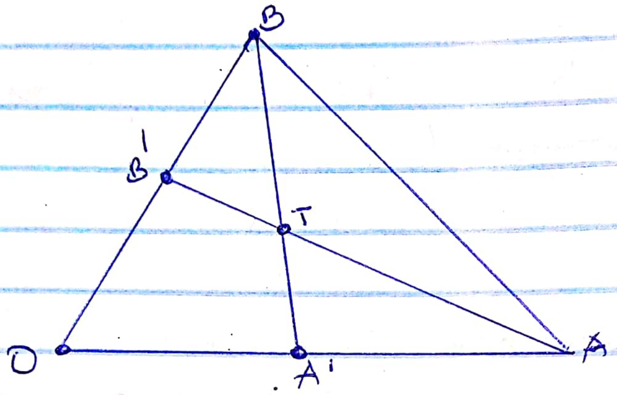
Q1 Incidence homomorphism from the Desargues configuration

Realization of the ...

Pappus configuration of plane $\mathbb{F}_3^2$.

Considering triangle OAB below:



The triangle has proportions when no explicit coordinates are given. Lets determine in which proportions the intersection of the midpoint bisector of a triangle divide the bisector

A' and B' are the midpoints of OA and OB respectively
To determine intersection T of AB' and BA' we interpret A, A' and B, B' as vectors with O as origin of our co-ordinate system

Then $T = (1-s)A + sB' = (1-t)B + tA'$

as $A' = \frac{1}{2}A$ and $B' = \frac{1}{2}B$.

$\Rightarrow (1-s)A + s(\frac{1}{2}B) = (1-t)B + t\frac{1}{2}A$

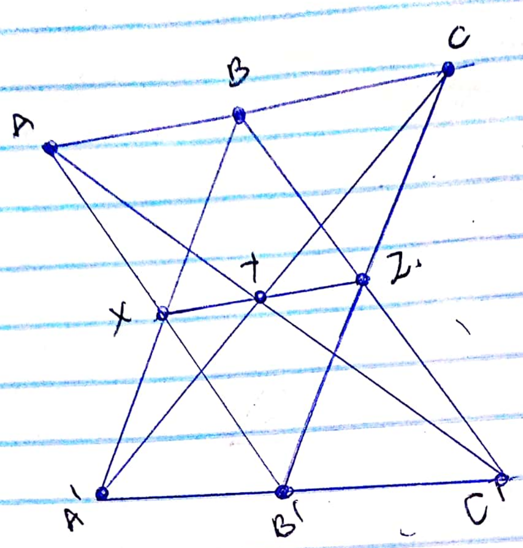
Regrouping gives $(1-s-\frac{1}{2}t)A = (1-t-\frac{1}{2}s)B$

As the vectors A and B are linearly independent, this implies that

$(1-s-\frac{1}{2}t) = 0 = 1-t-\frac{1}{2}v$

$\therefore s = t = \frac{2}{3}$ Hence, T divides BA' and AB' in the proportion 2 : 1

t and s are the same therefore there is incident homomorphism



Considering the set of 9 points, if we use the set $F_3 = \{0, 1, 2\}$ of integers of modulo 3. The affine plane F_3^2 consists of a 9 points that we can represent in a 3X3 array. There are 3 ~~paths~~ lines through any point, with a total of ~~12~~ 9 lines, each with 3 points on it.

A midpoint of a segment is always the third point on the line extending the segment. For triangle ABC in F_3^2, the vector from the midpoint $\frac{1}{2}(B+C) = -B-C$ to A is equal to $A+B+C$ and this remains true if midpoint bisectors ending at B or C are used. In other words we 3 midpoint bisectors of a triangle are parallel in geometry.

Q2: set of pairs $\{(a,b) : a, b \in \mathbb{F}_3\}$

$$(a,b) + (a'b') = (a+a', b+b')$$
$$(a,b) \cdot (a'b') = (aa' - bb', ab' + a'b)$$

$$1 = (1,0)$$
$$i = (0,1)$$

$$a + bi = (a,b)$$
$$i^2 = -1$$

The additive identity of $0 = 0 + 0i$, the multiplicative identity is $1 = 1 + 0i$ and the multiplicative inverse of a nonzero complex number $a + ib$ is $(a+bi)^{-1} = \frac{a}{a^2+b^2} + i(-b/(a^2+b^2))$

$$(a,b) + (a'b') = (a+a', b+b')$$
$$(a,b) \cdot (a'b') = (aa' - bb', ab' + a'b) = aa' - bb' + i(ab' + a'b)$$

If C denotes the set of ordered pairs (a,b)
The complex conjugate of the complex number ~~a~~ $z = a + ib$ is the complex number $\bar{z} = a - ib$
If a' is a positive real number the symbol $(\sqrt{a}'$ will be used to denote the positive (real) square root of c. Also $\sqrt{0} = 0$
$z = a + ib$ is a nonzero number i.e. a^2+b^2 is a positive real number

Absolute value of the complex number $z = a + ib$ is $|z| = \sqrt{z\bar{z}} = \sqrt{a^2+b^2}$

NB: $z = a + ib$ to - viewing $z = a + ib$ as a point $(a,b) \in \mathbb{R}^2$. ~~If θ is the angle that this~~ segment joining $(0,0)$ and (a,b) is $\sqrt{a^2+b^2} = |z|$
the length of the line segment joining $(0,0)$ and (a,b) is $\sqrt{a^2+b^2} = |z|$.
If θ is the angle that this segment makes with the positive first co-ordinate axis. Therefore $z = |z| \cos\theta$ and $b = |z| \sin\theta$. Usual convention is that positive angles are measured counterclockwise.

$z = |z|(\cos\theta + i\sin\theta)$

$z = |z|\cos\theta + |z| i\sin\theta.$ but $z = (a,b)$

$\frac{z}{|z|} = \cos\theta + i\sin\theta.$ $\Rightarrow \frac{1}{z} = \frac{\bar{z}}{||z||^2}$ if $|z| = 1.$

$\frac{1}{z} = \bar{z}$. Thus the imaginary units have additive inverse as well multiplicative inverse and are equal.

Since the modular product of these two elements is non zero the set F_3 is a field

we do not divide by zero because this will given us an infinite field.